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Application of vibrational parameters to locate the solid/liquid interface during solidification and melting of metals

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A method to locate the solid/liquid interface with vibrational parameters during solidification is proposed for the first time. The sufficient difference in resistance to shear stresses between liquid and solid phases of metals and alloys permits the application of vibrational parameters to locate the interface in real time and in a situation during solidification. Based on the solidification theory, continuum mechanics, vibrational modal analysis and sensitivity analysis, the mechanical model and the dynamic equations of a typical Bridgman solidifying system have been established, the sensitivity of eigenvalues of the Bridgman system to the location of the solid/liquid interface has been derived, and the formulae concerned calculated. The experimental results are in good agreement with the computed ones.

1. Introduction

The solid/liquid interface during solidification depends on the isothermal plane temperature which equals the melting point of the solidifying material. From the point of view of macro mechanical properties, the solid/liquid interface can be defined as the interface between the solid and liquid phases of the solidifying material. Because the control of solidification depends on the control of solidifying parameters, such as the solid/liquid interface movement velocity and the temperature gradient, it is of great importance both in research and in the production of solidification to locate the solid/liquid interface in each situation and in real time.

At the present time, the method used to determine the position of the solid/liquid interface is metallography. It does not give satisfactory results to the requirement of real-time analysis and control in the field, because it relies on a postmorten autopsy. Since around 1965, ultrasonic pulse-echo techniques have been used to locate the solid/liquid interface during solidification and melting of metals [1]. These techniques are based on the acoustic impedance difference between solid and liquid phases. It is known that only the solid phase possesses shear strength. Hence, the dynamic properties of the solidifying system (including eigenvalues, eigenvectors) vary with the solid/liquid interface position during solidification. A sufficient difference in resistance to shear stress between solid and liquid phases of metals permits the application of vibrational parameters to determine the position of the solid/liquid interface in real time and in each situation.

2. The model

The Bridgman solidifying system used here consists of a crucible [2], a crystallizer and a specimen, as shown structurally in Fig. 1a. The bottom of the crucible which contains the specimen is tightly connected with the crystallizer. Such a system can be simplified into a model of a cylindrical shell with variable cross-section and partly filled with fluid, as shown in Fig. 1b. Compared with the general model of a shell containing fluid, it has two distinguishing features: first, part of the model, especially the solid/liquid interface, is always in a closed opaque high-temperature field; second, the model is a time-variant system, because its stiffness varies with the ratio of solid to liquid.

In order to locate effectively the solid/liquid interface during solidification with vibrational parameters, we analysed the transverse vibration of the model. Under an axisymmetric load, the fundamental equation of a cylindrical shell is greatly reduced. The transverse vibration equation, where Φ_r is the radial displacement, a is the shell radius, δ the shell thickness, ν Poisson's ratio, P_r the radial external load, E the Young's modulus, is

$$R\frac{\partial^4\Phi_r}{\partial z^4} + \frac{E\delta}{a^2}\Phi_r = P_r, \tag{1a}$$

$$R = \frac{E\delta^3}{12(1-\nu^2)} \tag{1b}$$

The molten metal, i.e. the interior fluid, is considered as ideal and incompressible. The crucible in the Bridgman method does not revolve, so that the forced convection can be neglected. In addition, the direction of the temperature gradient is opposite to that of gravity and the density of the molten metal in the lower part of the crucible is heavier than that in the upper part, so natural convection is not taken into

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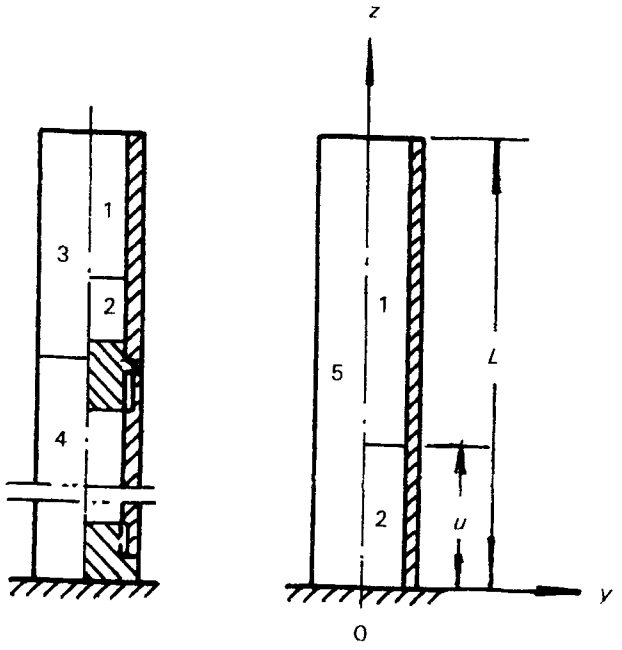

Figure 1 The Bridgman solidifying system. 1, Liquid; 2, solid; 3, crucible; 4, crystallizer; 5, cylindrical shell.

account either. According to a previous study on coupled vibration of a cylindrical shell filled with fluid [3], the effect of the sway of the molten metal can be neglected. Taking these points into consideration, the Bridgman solidifying system is regarded as a "liquid-solidified" elastic cylindrical shell. Its transverse vibration can be dealt with as a transverse vibration of a non-uniform beam. The centrifugal force, the Coriolis force, the axial force caused by temperature and internal pressure, the effect of shear force, rotating force, and inertia and damping are all neglected. The transverse free-vibration equation of the model (Equation 2), where E is the Young's modulus, I is the moment of inertia of the cross-section, ρ the mass density, A the area of cross-section, t the time, $y(z, t)$ a function of the transverse vibration, subscripts w, s, L represent wall of the shell, solid phase, and liquid phase respectively, is

$$(E_W I_W + E_S I_S)\frac{\partial^4 y(z,t)}{\partial z^4} + (\rho_W A_W + \rho_S A_S + \rho_L A_L)\frac{\partial^2 y(z,t)}{\partial t^2} = 0 \quad (2)$$

3. Sensitivity analysis of modal parameters

During solidification, the variation of the length ratio of solid to liquid phases in the crucible results in a corresponding variation of the modal parameters of the Bridgman solidifying system. We see [4] that the sensitivity of eigenvectors with respect to structural parameters can be expressed by the product of normal modes. Owing to the limitations of the Bridgman furnace, it is impossible to measure the normal modes. Therefore, the eigenvectors cannot be adopted to locate the solid/liquid interface, nor can modal mass, modal stiffness and modal damping. By contrast, it is possible to detect the natural frequency outside the furnace during solidification. The emphasis on analysis is given to the sensitivity analysis of eigenvalues, i.e. the relationship between natural frequency and the position of the solid/liquid interface. Perhaps the simplest approach to the solution of eigenvalues is to use Ritz's method. The boundary conditions are

$$y(0, t) = 0, \partial y/\partial z|_{z=0} = 0, \quad (3)$$

$$\left.\frac{\partial^2 y}{\partial z^2}\right|_{z=L} = \left.\frac{\partial^3 y}{\partial z^3}\right|_{z=L} = 0 \quad (4)$$

Because only the fundamental frequency is required, the function satisfies all boundary conditions.

$$Z = 6L^2 z^2 - 4Lz^3 + z^4 \quad (5)$$

$$\begin{aligned} U_{max} &= \int_0^u (E_W I_W + E_S I_S)(Z'')^2 \, dz \\ &\quad + \int_u^L E_W I_W (Z'')^2 \, dz \\ &= \frac{9\pi E_W}{20}\{(D^4 - d^4)L^5 \\ &\quad + \frac{E_S}{E_W}[d^4(L-u)^5 - d^4 L^5]\} \end{aligned} \quad (6)$$

$$\begin{aligned} T_{max} &= \int_0^u \rho_S A_S Z^2 \, dz + \int_u^L \rho_L A_L Z^2 \, dz \\ &\quad + \int_0^L \rho_W A_W Z^2 \, dz \\ &= \frac{\pi(D^2 - d^2)}{4}\rho_w(104/45)L^9 + \frac{\pi d^2}{4}\rho_S \\ &\quad (7.2u^5 L^4 - 8u^6 L^3 + 4u^7 L^2 - u^8 L \\ &\quad + u^9/9) + \frac{\pi d^2}{4}\rho_L[(104/45) - 7.2u^5 L^4 \\ &\quad + 8u^6 L^3 - 4u^7 L^2 + u^8 L - u^9/9] \end{aligned} \quad (7)$$

Thus, we obtain the approximate fundamental frequency as

$$\begin{aligned} \omega^2(r) &= \frac{1.8 E_W D^2}{\rho_w L^4}\{[1 - q^4(1-r)^5]\}/ \\ &\quad \{(104/45)[1 + (104/45)(\zeta - 1)q^2(104/45 \\ &\quad - 7.2r^5 + 8r^6 - 4r^7 + r^8 - r^9/9)]\} \end{aligned} \quad (8)$$

where ζ is the ratio of liquid density to crucible wall density, q is the ratio of the inner diameter, d, to the outer diameter, D, of the crucible, L is the total length, u the length of solid phase, r the ratio of u to L (including crucible and crystallizer).

Taking the initial state of the solidifying system ($r = 0$) as a base line, we obtain the dimensionless ratio of frequency

$$\begin{aligned} \lambda &= \lambda(r) = \omega(r)/\omega(0) \\ &= \{[1 - q^4(1 - r^5)][1 + (\zeta - 1)q^2]\}^{\frac{1}{2}} \\ &\quad \{(1 - q^4)[1 + 45/104)(\zeta - 1)q^2 \\ &\quad (104/45 - 7.2r^5 + 8r^6 - 4r^7 \\ &\quad + r^8 - r^9/9)]\}^{-\frac{1}{2}} \end{aligned} \quad (9)$$

Let S denote the sensitivity of the ratio λ (i.e. frequency) to the ratio r (i.e. length of solid phase)

$$S = S(\omega/r) = \frac{\partial\lambda}{\partial r} = \frac{1}{\omega(0)}\frac{\partial\omega(r)}{\partial r} \quad (10)$$

$$\frac{\partial\omega(r)}{\partial r} = \frac{1.8E_W D^2}{\rho_w 2\omega L^4}\{[5q^4(1-r)^4][(104/45) + (\zeta-1)q^2(104/45 - 7.2r^5 + 8r^6 - 4r^7 + r^8 - r^9/9)] - [1 - q^4(1-r)^5][q^2(\zeta-1)(-36r^4 + 48r^5 - 28r^6 + 8r^7 - r^8)]/[(104/45)(\zeta-1)q^2(104/45 - 7.2r^5 + 8r^6 - 4r^7 + r^8 - r^9/9)]^2\} \quad (11)$$

$$S = 0\ \partial\omega/\partial r = 0 \quad (12)$$

Thus

$$5q^2(1-r)^4[(104/45) + (\zeta-1)q^2(104/45 - 7.2r^5 + 8r^6 - 4r^7 + r^8 - r^9/9] - [1 - q^4(1-r)^5](\zeta-1)(-36r^4 + 48r^5 - 28r^6 + 8r^7 - r^8)] = 0 \quad (13)$$

Because

$$S|_{r=0} = \omega'(0)/\omega(0) = \frac{1.8E_W(104/9)[D^2 + (\zeta-1)d^2]q^4}{\rho_W 2\omega L^4[(104/45) + (\zeta-1)q^2(104/45)]^2} > 0 \quad (14a)$$

$$S|_{r=1} = \omega'(1)/\omega(0) = \frac{1.8E_W 9D^4(\zeta-1)d^2}{2\rho_W\omega L^4(104/45)^2} < 0 \quad (14b)$$

furthermore, S is a continuous elementary function.

Thus, $\vee r \in [0, 1]$, $\partial\lambda/\partial r = 0$, $\lambda|_{r=r_m} = \lambda_{max}$, $S|_{r=r_m} = 0$, that is, at the standing point $r = r_m$, though λ reaches its extreme value the vibrational parameters are useless in the identification of the solid/liquid interface. Because $\lambda = \lambda(r)$ is a higher degree polynomial of r, λ_{max} is a global extreme value, and the variation of function S is not small, we can apply vibrational parameters to locate the solid/liquid interface in spite of the existence of a standing point. It is imperative to prove that $\lambda = \lambda(r)$ is a normal convex function within the range of r.

From the point of view of mathematics, it is a non-linear programming with restraint, i.e. under the restraint

$$g_j(x_1, x_2, \ldots, x_m) \geqslant 0 \quad (j = 1, 2, \ldots, m)$$

For finding the optimum solution $(\bar{X}_1, \bar{X}_2, \ldots, \bar{X}_n)$, we make the non-linear objective function $f(X_1, X_2, \ldots, X_n)$ which is equal to its minimum (or maximum) $f(\bar{X}_1, \bar{X}_2, \ldots, \bar{X}_n)$. Hence the resulting function is $\lambda = \lambda(r) = f_1(r)/f_2$, a fractional function.

According to the theorem of convex function and generalized convexity, the convexity of $\lambda = \lambda(r)$ depends on its numerator $f_1(r)$, another linear fractional function, as its non-negative denominator f_2 is independent of r. The numerator $f_1(r)$ can be further divided into numerator and denominator parts. Because the numerator part is a non-negative concave function, the denominator part is a positive convex function, thus, $f_1(r)$ is a pseudo-concave function and λ_{max} is a global extreme value. Let $q = 0.9$, $\zeta = 0.45$, $r = ih$, $i = 1, 2, \ldots, 100$, $h = 0.01$. To find the solution of Equation 13 with a numerical method, such as Newton's method, we obtain $r_m = 0.4564$, $\lambda(0.4564) = \lambda_{max} = 1.653$. The computed results are shown in Table I, and the corresponding curve of $\lambda(r)$ is shown in Fig. 2. Fig. 2 shows that the sensitivity of natural frequency, S, is the slope of the tangent of a point on the curve. It can be noted that S neither remains constant nor changes greatly. In accordance with its magnitude, S can be classified into three categories: "sensitive", "blind" and "dull", during solidification. In application, S can fall into the "sensitive" region by adjusting structural parameters, e.g. the ratio of the solid phase to the liquid phase.

4. Experimental procedure

Fig. 3 shows the structure of the furnace, the excitation and detection instruments for the tests made on Al–3% Mg alloy. The arrangement is basically a modified Bridgman crystal growth furnace design, which is chosen because it allows unidirectional solidification and melting with a virtually planar interface, controlled solidifying parameters and fast liquid

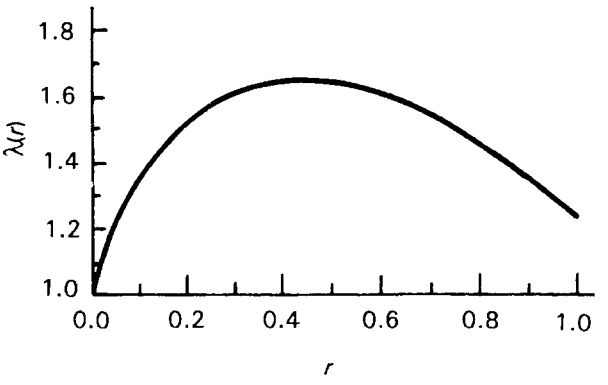

Figure 2 The curve of $\lambda(r)$.

TABLE I Numerical results of Equation 4

r	λ	r	λ	r	λ	r	λ
0.00	1.000	0.26	1.574	0.52	1.645	0.78	1.488
0.02	1.087	0.28	1.590	0.54	1.640	0.80	1.469
0.04	1.162	0.30	1.604	0.56	1.634	0.82	1.449
0.06	1.227	0.32	1.616	0.58	1.626	0.84	1.429
0.08	1.284	0.34	1.627	0.60	1.617	0.86	1.409
0.10	1.334	0.36	1.635	0.62	1.607	0.88	1.387
0.12	1.378	0.38	1.642	0.64	1.596	0.90	1.366
0.14	1.417	0.40	1.647	0.66	1.584	0.92	1.344
0.16	1.452	0.42	1.650	0.68	1.570	0.94	1.322
0.18	1.483	0.44	1.652	0.70	1.555	0.96	1.300
0.20	1.510	0.46	1.652	0.72	1.540	0.98	1.278
0.22	1.534	0.48	1.652	0.74	1.523	1.00	1.255
0.24	1.555	0.50	1.649	0.76	1.506		

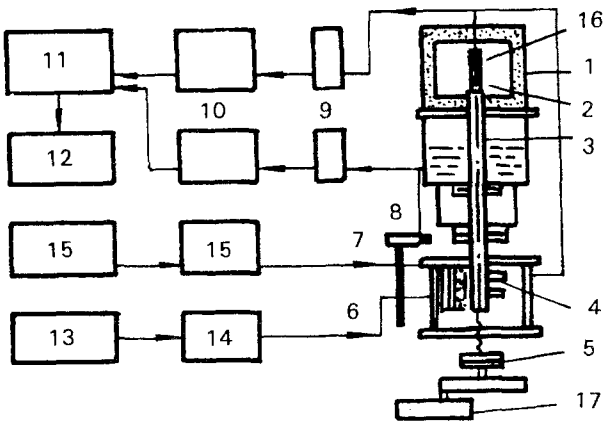

Figure 3 Test instruments. 1, Furnace; 2, specimen; 3, crystallizer; 4, accelerometer; 5, transfer machine; 6, magnetoelectric exciter; 7, hammer; 8, force transducer; 9, charge amplifiers; 10, voltmeter; 11, signal analyser, 12, X–Y plotter; 13, signal generator; 14, power amplifier; 15, d.c. power supply; 16, crucible; 17, electromotor.

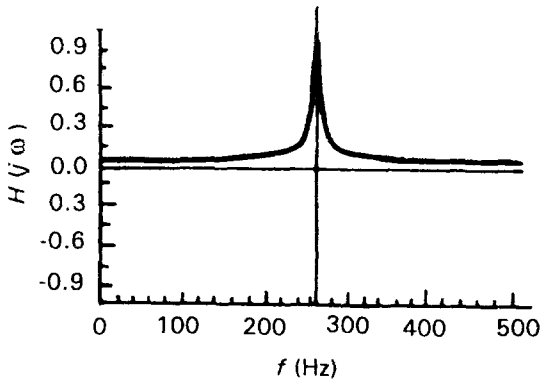

Figure 4 Frequency response function of the solidifying system.

quenching. The alundum crucible was heated by an electrical resistance furnace. During solidification, the position of the solid/liquid interface can be altered through adjustment of the level of cool water.

Several types of excitation were tested (random, pseudo-random, etc.). Impulse tests were carried out with a dual-channel signal processing system chiefly composed of an IBM-PC which has a zoom function to obtain model parameters. At the beginning of the solidification, while the level of cool water was at its lowest position, the Al–3% Mg alloy specimen was heated to partial melting and a steady thermal field was established both in the furnace and in the crucible.

Then, the solidifying system was excited by hammer impulse at a point on an attainable section of the crystallizer, and simultaneously, the response of the system was picked up by the accelerometer. The sampling time was 1 ms. Both force and response signals were sent into the processing system and analysed. Fig. 4 shows the magnitude of the frequency response function in coincidence with a given position of the solid/liquid interface. Modal parameters can be identified through the frequency response function. In this paper eigenvalues are identified.

After exciting, sampling and analysing, the position of the solid/liquid interface was measured directly using a piece of an alundum probe inserted into the

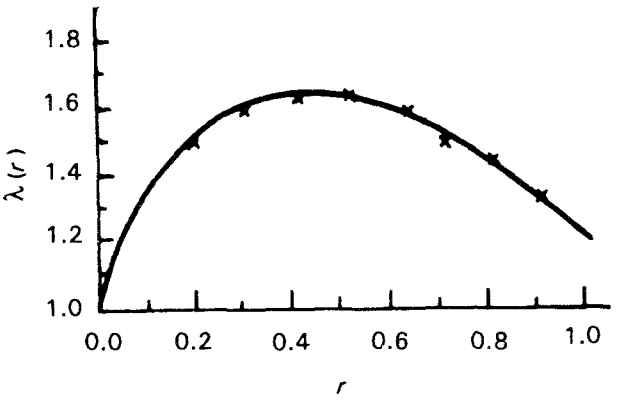

Figure 5 (×) Measured and (—) computed values of $\lambda(r)$.

TABLE II A comparison of the measured frequency ratio, λ, and natural frequency, f, with computed ones under different ratios of length of solid phase to total length

Test	r (length ratio)	λ		f(Hz)	
		Measured	Computed	Measured	Computed
1	0.31	1.60	1.614	281.2	282.4
2	0.40	1.63	1.645	285.1	288.3
3	0.50	1.67	1.649	292.2	288.7
4	0.60	1.62	1.618	283.2	283.1
5	0.70	1.55	1.555	271.4	272.3
6	0.81	1.48	1.459	259.7	257.1
7	0.92	1.40	1.344	246.0	235.0

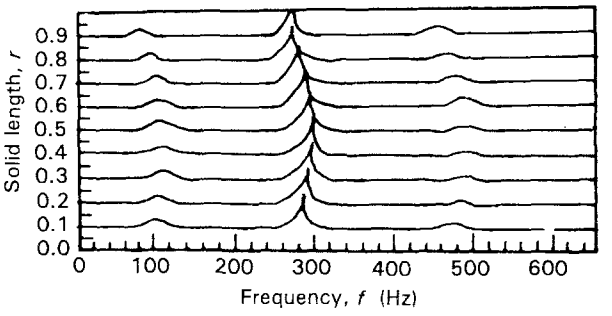

Figure 6 Relationship between the position of the solid/liquid interface and the response power spectrum.

crucible and by using a gauge. Thus, a pair of r and λ values was obtained. Then all conditions were held constant, except the level of cooling water. The solid phase of the specimen becomes longer when the level is raised to the desired height. Through meter monitoring, it is clear whether or not the steady thermal field is set up. As long as the field becomes steady again, the experimental procedures will be repeated. Therefore, another pair of r and λ values is obtained. By repetition of the procedure, we acquire several pairs of r and λ values. The experimental results and the computed results are listed in Table II. A comparison is shown in Fig. 5.

The relationship between the position of the solid/liquid interface and the response power spectrum is shown in Fig. 6, the response auto-spectrum is given which corresponds to different positions of the solid/liquid interface. The frequency coinciding with

a peak value of the spectrum line is the eigenvalue corresponding to a given ratio of solid length to total length. In practice, the map of spectral sequences at different moments can be recorded by an X–Y plotter and used to locate the solid/liquid interface in real time.

5. Conclusions

1. It has been demonstrated by experiment and calculation that the position of the solid/liquid interface can be determined from vibration parameters during solidification.

2. In view of the characteristics of the Bridgman solidifying system, we adopted the sensitivity of the transverse eigenvalue with respect to the position of the solid/liquid interface, to locate the interface outside the furnace in real time.

3. The work reported here gives the computed formula and relevant experimental detection methods of sensitivity in a Bridgman system. In identification, sensitivity may be "sensitive" or "blind".

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